JHEP HYPER VERSION

Received: *July 31, 2001,* Accepted: *September 1, 2001*

Fermions in bosonic string theories

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Abstract: We generalize the Jackiw-Rebbi-Hasenfratz-'t Hooft construction of fermions from bosons to demonstrate the fermionic nature of certain bound states involving $\mathrm{SU}(N)$ instantons in even spatial dimensions and $\mathrm{SO}(N)$ instantons in $8k+1$ spatial dimensions. We use this result to identify several fermionic excitations in various perturbatively bosonic string theories. In some examples we are able to identify these fermions as excitations in known conformal field theories and independently confirm their fermionic nature. Examples of the fermions we find include certain 3-string junctions in type-0B theory, excitations of the 0-p system in type-0A theory, excitations of the stable D-particle of type-0 theory, and a rich spectrum of fermions in the bosonic string compactified on the SO(32) group lattice.

Keywords: Solitons Monopoles and Instantons, Bosonic Strings, D-branes, String Duality.

JHEP09(2001)001

Contents

1. Introduction

Over the last few years the various apparently distinct superstring theories have been identified as different corners on the moduli space of a single theory. However the relationship of the bosonic string, perhaps the simplest string theory, to M-theory is still unclear.

The bosonic string is qualitatively different from its supersymmetric cousins in two important respects; it is tachyonic and it does not contain space time fermions in its perturbative spectrum. However, both these features are also true of the type-0A string which, nonetheless, has recently found its conjectural position in the M-theory family. Type-0A theory has been interpreted as type-IIA theory with a background RR 2-form flux [1] (see also [2]) or, equivalently, M-theory compactified on a thermal circle [3, 1]. The type-0A tachyon is conjectured to signal the instability for this state to decay into supersymmetric type-IIA vacuum [2, 1]. These conjectures[1] are consistent with the purely bosonic nature of perturbative type-0A theory as the massless fermions of M-theory are driven out of the perturbative type-0A spectrum by the anti-periodic boundary conditions on the M-theory circle. However they reappear in the non-perturbative type-0A spectrum with energies of order $1/R_{11} = 1/g$ as excitations on D-branes [3].

It is natural to wonder if other perturbatively bosonic string theories, including the bosonic string, are connected to the M-theory mainland in a manner analogous to that conjectured for the 0A theory. In this paper we begin an investigation of this question. In particular we identify a spectrum of non-perturbative fermionic excitations in several bosonic string theories, including the bosonic string.

Over twenty five years ago Jackiw and Rebbi [6] Hasenfratz and 't Hooft [7] argued that certain purely bosonic SU(2) gauge theories possess nonperturbative fermionic excitations. As string theories often contain gauge fields, generalizations of the Jackiw-Rebbi-Hasenfratz-'t Hooft construction allow us to identify several nonperturbative fermions in perturbatively bosonic string theories.

As we review in section 2, Jackiw, Rebbi, Hasenfratz and 't Hooft argue that the bound state of an SU(2) 't Hooft-Polyakov monopole with a quantum that transforms in the fundamental of SU(2), is fermionic. It is rather amusing to note (see section 3 for details) that the familiar 3-string junction sketched in figure 1 is a precise implementation of the Jackiw-Rebbi-Hasenfratz-'t Hooft construction on the world volume of D3-branes in bosonic type-0B string theory and perhaps also in bosonic string theory. Consequently, the 3-string junction of figure 1 is a fermion!

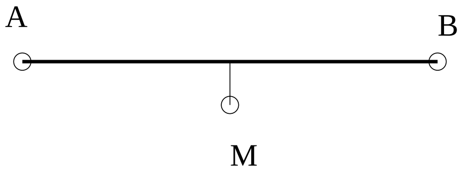

Figure 1: A 3-string junction. The fundamental string emerging from D3-brane M meets a D-string emerging from the D3-brane A to form a (1,1) string which ends on the D3-brane B. The D3-branes are parallel, but are separated in transverse space.

In section 4 of this paper we generalize the Jackiw-Rebbi-Hasenfratz-'t Hooft construction. We argue that, for sufficiently large N, that bound states involving instantons of SU(N) gauge theories in even spatial dimensions, and SO(N) gauge

[1] Similar conjectures have also recently been made relating nonsupersymmetric heterotic string theories to their supersymmetric counterparts [4, 5].

theories in $8k+1$ spatial dimensions, are fermionic. In particular, we demonstrate that bound states of the familiar SU(2) instanton with a fundamental quantum is a fermionic particle in $4+1$ dimensional SU(N) gauge theory.

In section 5, we employ these constructions to identify several fermions in peturbatively bosonic string theories. The fermions we study fall into two categories. Excitations of the first sort propagate on D-branes, and are bound states of perturbative quanta with nonperturbative lumps constructed out of D-brane gauge fields. In some of the examples that we study, it is possible to identify a conformal field theory associated with the nonperturbative lump, and thereby independently verify the fermionic nature of its bound states using worldsheet techniques.

Fermions in the second category are constructed out of gauge fields obtained from compactification. These excitations propagate in all the bulk noncompact directions. For example, the bosonic string compactified on the SO(32) group lattice[2] has a potentially rich spectrum of fermions.

In closing we note that Bergman and Gaberdiel [10, 3] and Klebanov and Tseytlin [11, 12] have pointed out several years ago that strings stretching between $|\mathrm{D}p,+\rangle$ and $|\mathrm{D}p,-\rangle$-branes, in type-0A and 0B theory, are fermionic. Such states appear to be fermions made out of bosons in a manner distinct from the constructions studied in this paper. It would be interesting to understand this better.

2. Spin from isospin

In this section we review the Jackiw-Rebbi-Hasenfratz-'t Hooft construction of fermionic excitations in a 3+1 dimensional bosonic gauge theory.

Consider an SU(2) gauge theory in 3+1 dimensions with a single real adjoint field φ. The action for this system is

$$S = \frac{1}{g_{\mathrm{YM}}^2} \int d^4x \,\mathrm{Tr}\left(-\frac{1}{2} F_{\mu\nu}F^{\mu\nu} - D^\mu \varphi D_\mu \varphi\right). \tag{2.1}$$

Eq. (2.1) may be spontaneously broken to U(1) by a constant φ vacuum expectation value, let $(\varphi^1)^2 + (\varphi^2)^2 + (\varphi^3)^2 = v^2$ in the vacuum. The 't Hooft-Polyakov monopole that asymptotes to this vacuum is

$$\begin{aligned} A_0^a &= 0 \\ A_i^a &= \frac{\epsilon_{ij}^a x^j W(vr)}{r^2} \\ \varphi^a &= \frac{x^a H(vr)}{r^2}\,, \end{aligned} \tag{2.2}$$

[2]The authors of [8] (see also [9] and references therein) have also studied the bosonic string compactified down to 10 dimensions group lattices with closely related motivations.

where $i = 1, \ldots, 3$ is a spatial vector index, $a = 1, \ldots, 3$ is an SU(2) isospin vector index and the radial functions $W(vr)$ and $H(vr)$ are given by

$$W(vr) = 1 - \frac{vr}{\sinh vr}, \qquad H(vr) = vr \coth vr - 1. \tag{2.3}$$

Note that (2.2) does not transform covariantly under either spatial rotations or global isospin rotations. However it does transform covariantly under a diagonal combination of the two. As we review below, this observation allows one to argue that the bound state of the monopole (2.2) with a particle transforming in a half integral representation of the gauge group SU(2) is fermionic.

Consider [7] a spinless nonrelativistic particle P, transforming in the R^{th} representation of SU(2), interacting with the monopole (2.2). In an approximation that ignores all propagating modes of the fields A_μ and φ, the system is described by a Schrödinger equation for P in the background (2.2). As pointed out above, (2.2) transforms covariantly under simultaneous spatial and isospin rotations. Consequently the hamiltonian for the motion of P in the background (2.2) commutes with the SU(2) generators

$$J^a = \epsilon^{abc} x^b p^c + T^a_R, \tag{2.4}$$

where x^b is the position operator, p^c its canonical conjugate, and T^a_R the isospin generator in the R^{th} representation. Consequently eigenstates of the hamiltonian appear in multiplets of the SU(2) generated by J. As states of the entire system, consisting of the monopole plus the particle, are independently expected to appear in SU(2) multiplets of total angular momentum, Hasenfratz and 't Hooft [7] proposed that J^a be identified with the net angular momentum of the system. As evidence for this proposal, Hasenfratz and 't Hooft have demonstrated that, classically, the angular momentum stored in the Yang-Mills field when a charged particle interacts with a Yang-Mills monople is indeed given by (2.4) (see appendix B). According to this proposal, the quantum numbers of (2.4) should be interpreted as spin quantum numbers for the monopole-charged particle bound state. Consequently, the bound state of a monopole with a particle in a half integral representation of the gauge group has half integral spin. By the spin statistics theorem, such a bound state must be fermionic.

The angular momentum and fermionic nature of this charge-monopole bound state may also be understood from the long distance U(1) description of this system, as we review in appendix A.

3. Fermionic 3-string junctions

As a concrete application of the mechanism reviewed in the previous section we will now argue that the 3-string junction described in the introduction, and sketched in figure 1 is a fermion on the world volume of the D3-branes that host it. The arguments of subsection 3.1 apply to the type-0B theory, type-IIB string theory and perhaps even the bosonic string. In subsections 3.2–3.4 we will separately examine the construction in each of these theories.

3.1 The 3-string junction as a charge-monopole bound state

Consider a D-string stretched between two parallel D3-branes A and B.

When the separation between the D3-branes is small in string units the configuration of figure 2 is well described as a monopole [13] in the SU(2) theory on the world volumes of branes A and B. Consider a string stretching between the branes A and B and the brane M. Such a mode transforms in the fundamental of SU(2). According to the arguments of the previous section, a bound state of this mode with the monopole is a fermion.

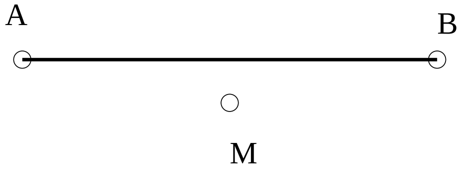

Figure 2: A D-string stretched between D3-branes A and B. The 3rd brane M is a spectator.

It is easy to argue for the existence of at least one such bound state at weak coupling [14]. From the viewpoint of string theory, the existence of a bound state is a consequence of the fact that, at weak coupling, flux dissolved inside a D-string has less energy than an F-string as $\sqrt{(\frac{1}{2\pi\alpha'})^2 + (\frac{1}{2\pi\alpha' g})^2} < \frac{1}{2\pi\alpha' g} + \frac{1}{2\pi\alpha'}$. This bound state is the 3-string junction of figure 1.

In the general case the 3-string junction between a $(p, 0)$, (r, q) and $(p + r, q)$ is bosonic if pq is even and fermionic if pq is odd. This prediction can, in principle, be checked by a direct computation of the angular momentum stored in the field of the corresponding solution in Yang-Mills theory.[3]

3.2 0B-theory

Type-0B theory has two varieties of stable D-branes of each dimensionality; the two flavors of D-branes are called $|\mathrm{D}p, +\rangle$ and $|\mathrm{D}p, -\rangle$-branes, and are distinguished by a sign in the fermionic part of the equation that defines the corresponding boundary state [22].[4] We will use the following important property several times in this paper: the open strings running between branes or antibranes of the same sign (both + or both −) are always in the Neveu-Schwarz sector, whereas strings stretching between branes or antibranes of different signs are always in the Ramond sector [3].

[3] Several authors have investigated solutions of Yang-Mills theory corresponding to quater BPS states [15, 16] present a rather explicit construction of some 3-string junction solutions for special values of p, q and r [17]. However, for each of these special values, pq is even: the corresponding solutions are radially symmetric and so have zero angular momentum. General 3-string junctions are formally constructed in [18], where it is pointed out that the generic 3-string junctions is not radially symmetric. In fact, the 3 string junction should be thought of as a multi centred solution [19, 20, 21].

[4] For a recent review on type-0A and 0B theories, their D-branes see [23]. We will use the notation of that paper.

Let A and B in figure 2. both represent $|\mathrm{D3},+\rangle$ branes occupying spatial dimensions $1,2,3$. If follows from the 3-brane world volume coupling [24] proportional to $\int C^{+}_{0i}B_i$, where B_i is the worldvolume magnetic field, that lines of magnetic flux on a $|D3+\rangle$-brane carry $|\mathrm{D1}+\rangle$ charge. Consequently, the SU(2) monopole on these 3-branes represents a a $|\mathrm{D1},+\rangle$-brane stretching between A and B, in, say the 4 direction. If M is also a $|\mathrm{D3},+\rangle$-brane, as in section 4.1, the 3 string junction is a stable fermionic bound state of purely bosonic degrees of freedom propagating on the $|\mathrm{D3},+\rangle$-branes.

The fermionic nature of this 3-string junction may also be understood from the point of view of the string worldsheet. According to the rules stated above, the fundamental string from M to the $|\mathrm{D1},+\rangle$-brane is in the Neveu-Schwarz sector. Neveu-Schwarz sector fermions in Neumann-Dirichlet directions (in this case $\psi^1,\psi^2,\psi^3,\,\psi^4$) are integrally moded; in particular the mode expansion for each of these fermions includes a zero mode. However the x^4 coordinate at the ends of the D-string (on A and B) are fixed and it is plausible these boundary conditions project out the ψ^4 zero mode, but retain the other three. In which case all open string states appear in representations generated by the ψ^1,ψ^2,ψ^3 zero mode algebra, i.e. are SO(3) spinors.

On the other hand, if M is a $|\mathrm{D3},-\rangle$-brane, strings from M to A or B are fermionic [3]. The 3 string junction of figure 1 is then a bound state of the monopole with a fermion in the fundamental of SU(2), and is consequently a boson! From a worldsheet viewpoint this is simply a consequence of the fact that Ramond sector fermions in Neumann-Dirichlet directions, (in particular ψ^1,ψ^2,ψ^3), are half integrally moded, and so do not possess a zero mode.

3.3 IIB-theory

3-string junctions in IIB theory appear in $1/4^{\mathrm{th}}$ BPS multiplets of the $\mathcal{N}=4$ super algebra on the world volume of the 3-branes. The assignment of half integral spins to the purely bosonic states in this multiplet is consistent with this super multiplet structure, as we explain in appendix C using the analysis of [18] (see also [25]).

3.4 The bosonic string

The situation is much more confusing in the Bosonic String. At least at leading order in α', the open string tachyon couples to gauge and scalar fields only quadratically, and so may consistently be set to zero. Consequently, when all brane separations are small in string units, the SU(2) monopole with the tachyon set to zero is an approximate solution of the equations of motion on the D3-branes. The bound state of this monopole with a fundamental quantum, described in section 3.1, is a fermionic excitation in bosonic string theory.

However, the interpretation of the monopole as a stretched D-string, and correspondingly the above configuration as a 3-string junction is confusing, and may be incorrect[5] in the bosonic string. The energy of the monopole matches the expected energy of a stretched D1-brane. Further the low energy dyonic excitations of the monopole match the flux excitations expected of stretched D1-branes. However the monopole is charged, and in particular appears in two varieties, whereas bosonic D-strings are unoriented and uncharged.

It is possible that the end points of D-strings on D3-branes are charged, even though the the D-strings themselves are uncharged. That certainly seems to be true of non-BPS-branes in type-IIA theory.[6] Consider a D-string and a coincident anti D string ending on a 3-brane- anti 3-brane system in IIB theory. The D-string could end on either the 3-brane or the anti 3-brane, and the same is true of the anti D-string. Of these four possible boundary conditions, the two configurations in which both D-strings end on the same 3-brane are not invariant under $(-1)^{F_L}$ under which D-branes and anti D-branes are interchanged. The remaining two boundary conditions are invariant under $(-1)^{F_L}$, and so can be modded out by $(-1)^{F_L}$. Consider the configuration in which the D-string ends, with a positive magnetic charge on the 3-brane, and the anti D-string ends, also with a positive magnetic charge,[7] on the anti 3-brane. Orbifolding this configuration by $(-1)^{F_L}$ gives a non BPS D-string ending on the non-BPS 3-brane of IIA theory. The end point of this D-string has positive magnetic charge under the centre of mass U(1) that survives the orbifold projection. Orbifolding the other $(-1)^{F_L}$ invariant configuration (the D-string ending on the anti 3-brane, and the anti D-string ending on the 3-brane) leads to a non-BPS D-string ending on a non-BPS 3-brane with a negative magnetic charge.

Thus uncharged and unoriented non-BPS D-strings can end on non-BPS 3-branes in two inequivalent ways. These endpoints are associated with either positive or negative magnetic charge in the 3-brane theory. It would be interesting to understand if this was true even in the bosonic string theory.

Another confusing question surrounding this configuration is the following: what happens to this fermion, after the D3-branes that host it have disappeared due to tachyon condensation? This question has a perhaps sharper analogue in type-0A theory. Consider an unstable 3-brane of the + variety, $|\widehat{\mathrm{D3}}, +\rangle$, joined to an unstable 3-brane of the $-$ variety, $|\widehat{\mathrm{D3}}, -\rangle$ by a single fundamental string. What happens to this fermionic excitation after the two unstable 3-branes have decayed due to tachyon condensation? In both cases the resulting configuration would appear to be an infinitely long open string with fermionic nature. It would be interesting to understand this better.

[5]We would like to thank the participants of the Amsterdam workshop for emphasizing this to us.
[6]We thank Ashoke Sen for explaining this to us.

[7]The Chern-Simons couplings on antibranes have a relative minus sign compared to those on D-branes. Consequently an anti D-string ends on an anti 3-brane with the same charge as a D-string on a 3-brane.

4. Spin from isospin in higher dimensions

As reviewed in section 2, a bound state of an SU(2) monopole and a fundamental quantum in 3+1 dimensions should be regarded as a fermion. In four or higher dimensions finite energy solitons analogous to the 't Hooft-Polyakov monopole do not exist.[8] However instantons of the d dimensional gauge theory are localized finite energy field configurations. These configurations are classified by $\pi_{d-1}(G)$, where G is the gauge group and are stabilized by the winding of the gauge group at infinity.

In this section we will study instantons of $\mathrm{U}(N)$ and $\mathrm{SO}(N)$ gauge theories, and argue that their bound states with quanta transforming in certain representations of the gauge group are often fermionic, generalizing the mechanisms for producing spin from isospin reviewed in section 2.

While instantons are scale invariant in 4 dimensions, energetics drives them to shrink to a point in higher dimensions. In the applications we have in mind, we expect these instantons to be stabilized at short distances by stringy effects. Since all our conclusions will follow from long distance topological considerations, the details of this stabilization will not bother us.

4.1 Fermions from $\mathrm{U}(N)$ instantons

The even homotopy groups of $\mathrm{U}(N)$ for $N \geq 2$ are trivial ($\pi_i(\mathrm{SU}(N)) = 0$ for all even $i \geq 2$) while the odd homotopy groups are integer valued ($\pi_i(\mathrm{SU}(N)) = Z$ for all odd $i \geq 3$). Consequently interesting $\mathrm{U}(N)$ instantons exist on even dimensional spaces; the quantized instanton number of a configuration in $d = 2m$ dimensions is given by $\frac{1}{m!(2\pi)^m} \int F \wedge F \wedge, \ldots, \wedge F$.

Note also that $\mathrm{U}(N)$ homotopy groups are stable for sufficiently large N. Consequently, for N sufficiently large compared to d a $\mathrm{U}(N+1)$ instanton is isomorphic to the trivial embedding of a $\mathrm{U}(N)$ instanton into $\mathrm{U}(N+1)$.

In this subsection we study $\mathrm{U}(N)$ instantons in d spatial dimensions, where d is even. The stability property of $\mathrm{U}(N)$ homotopy groups permits us to study any particular convenient large value of N for every spatial d. Most of our results will then be easy to generalize to other large values of N. We find it convenient to choose $N = 2^{\frac{d-2}{2}}$ as this allows us to use the Atiyah Bott Shapiro construction [26] (see also [27]) of this instanton at infinity.

Consider an $\mathrm{SU}(2^{\frac{d-2}{2}})$ theory in d (even) spatial dimensions. Let γ_i ($i = 1, \ldots, d-1$) represent the $2^{\frac{d-2}{2}} \times 2^{\frac{d-2}{2}}$ $\mathrm{SO}(d-1)$ Γ matrices. Define $\gamma_d = -iI$. Consider an

[8] This may be understood as follows. Consider a scalar field (charged under the gauge group) that winds at infinity. The angular derivatives of this field decay at infinity like $1/r$; in order that all covariant derivatives decay sufficiently rapidly, the gauge field is forced to take a specific value at $\mathcal{O}(1/r)$, implying $F \sim \mathcal{O}(1/r^2)$, and a configuration energy $E \approx \int d^d x F^2 \sim \int d^d x/r^4$, divergent for $d \geq 4$, where d is the spatial dimension.

instanton in this theory whose gauge field takes the form

$$A_\mu = -i\partial_\mu U U^{-1} f(x^2)\,, \tag{4.1}$$

where

$$U(x) = \frac{\gamma_\mu x^\mu}{|x|} \tag{4.2}$$

and f tends to unity as its argument becomes large. $(\gamma_\mu)_{\alpha\dot\alpha}$ is an SO(d) invariant,[9] where where α is a chiral SO(d) spinor index and $\dot\alpha$ is an antichiral SO(d) index. Therefore, under a spatial rotation R,

$$U \to S(R) U \bar{S}^T(R)\,, \tag{4.3}$$

where $S(R)$ and $\bar{S}(R)$ respectively are the chiral and antichiral spinorial matrices corresponding to the rotation R. Consequently, under the same rotation,

$$A_\mu \to S(R) R_\mu^\nu A_\nu S^{-1}(R)\,. \tag{4.4}$$

Thus, A_μ transforms covariantly under the rotation R accompanied by a simultaneous large gauge transformation by the $U(2^{\frac{d-2}{2}})$ matrix $S(R)$. Consequently, as in section 2, the conserved angular momentum $J_{\mu\nu}$ of the system is

$$J_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu + T_{\mu\nu}\,, \tag{4.5}$$

where $T_{\mu\nu}$ are the generators of an SO(d) subgroup of SU($2^{\frac{d-2}{2}}$).

As an immediate consequence, the bound state of a fundamental scalar with this instanton is a chiral spinor, and so is fermionic. This state pairs up with the corresponding anti-instanton bound state, an antichiral spinor, to form a full $(d,1)$ dimensional spinor.

Several remarks are in order.

1. The SO(d) subgroup of SU($2^{\frac{d-2}{2}}$) that appears in (4.5), is determined by the following defining property: the fundamental representation of SU($2^{\frac{d-2}{2}}$) transforms as a chiral spinor of SO(d). In general an arbitrary representation of SU($2^{\frac{d-2}{2}}$) will decompose into several representations of SO(d). This decomposition may be determined as follows. The $\frac{d}{2}$ Cartan generators of SO(d) may be determined as functions of the SU($2^{\frac{d-2}{2}}$), as the explicit form of generators of both groups is available in the $2^{\frac{d-2}{2}}$ dimensional representation. Consequently the SO(d) weights, and representation content of the bound state with any SU($2^{\frac{d-2}{2}}$) quantum may be determined.

[9]This follows from the observation that the set of matrices $\Gamma_i \otimes \sigma_1, I \otimes \sigma_2$ constitute a set of SO(d) Γ matrices whose chirality matrix is $I \otimes \sigma_3$.

2. As (4.1) is pure gauge at infinity, the instanton does not break any gauge symmetry unlike the monopole (2.2). At infinity, A_μ in (4.1) may be locally set to zero in coordinate patches. Consequently, the bound state of an instanton with a particle in the R^{th} representation of U(N) continues to transform in the R^{th} representation of the unbroken U(N).

3. As remarked at the beginning of this subsection, it is straightforward to generalize the considerations of this subsection to SU(N) theories with $N > 2^{\frac{d-2}{2}}$. The instantons of this subsection may simply be embedded into a block diagonal SU($2^{\frac{d-2}{2}}$) subgroup of SU(N).

4. It is instructive to separately study the simplest special case, namely the familiar SU(2) instantons in $d = 4$. Eq. (4.2) takes the form

$$U(x) = \frac{\sigma_\mu x^\mu}{|x|}\,, \tag{4.6}$$

where x^μ, ($\mu = 1, \ldots, 4$) are the spatial coordinates, σ_i are $i = 1, \ldots, 3$ are the Pauli matrices, and $\sigma_4 = -iI$. The rotational symmetry group of R^4 is $\mathrm{SO}(4) = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Repeating the general arguments of this subsection, we find that the true angular momentum generators are

$$K'_a = K_a + T_a\,, \qquad L_a\,, \qquad a = 1, \ldots, 3\,, \tag{4.7}$$

where T_a are the SU(2) gauge generators, and K and L are the generators of $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$.[10] In the classical approximation, it is possible to independently compute the angular momentum stored in the Yang-Mills field for the configuration consisting of a particle in the R^{th} representation of SU(2) placed in the background of a Yang-Mills instanton. We present the relevant computation in appendix B; we find agreement with (4.7).

4.2 SO(N) instantons

In this subsection we will study bound states of charged quanta with SO(N) instantons in $8k + 1$ spatial dimensions, and find that several of these are fermionic.

$\pi_{8k}(\mathrm{SO}(N)) = Z_2$ for $N \geq 8k + 2$; consequently SO(N) theories in $8k + 1$ spatial dimensions possess a unique nontrivial instanton. Consider, for concreteness, an SO(16) gauge theory in 9+1 dimensions. Recall that the 9 16×16 Γ matrices Γ_μ of

[10] Specifically, K and L are linear combinations of the usual SO(4) generators

$$\begin{aligned} K_a &= \frac{\frac{1}{2}\epsilon_{aij}L_{ij} + L_{0a}}{2} = \frac{1}{4}\eta^{a\mu\nu}L_{\mu\nu} = \frac{1}{2}\eta^{a\mu\nu}x_\mu p_\nu \\ L_a &= \frac{\frac{1}{2}\epsilon_{aij}L_{ij} - L_{0a}}{2} = \frac{1}{4}\bar{\eta}^{a\mu\nu}L_{\mu\nu} = \frac{1}{2}\bar{\eta}^{a\mu\nu}x_\mu p_\nu\,, \end{aligned} \tag{4.8}$$

where $\eta^{a\mu\nu}$ and $\bar{\eta}^{a\mu\nu}$ are the 't Hooft symbols defined and described in equations A5-A15 of [28].

SO(9) may be chosen to be real and symmetric; we make such a choice. The unique instanton may be put into the form[11]

$$A_\mu = -i\partial_\mu O O^{-1} f(x^2)\,, \qquad O(x) = \frac{\Gamma_\mu x^\mu}{|x|.}\,, \tag{4.9}$$

where $O(x)$ is an SO(16) group element, and $\lim_{x^2\to\infty} f(x^2) = 1$. $(\Gamma_\mu)_\alpha^\beta$ is an SO(9) invariant where μ is a vector index and α and β are SO(9) spinor indices. Imitating the logic of the previous subsections we conclude that the angular momentum operator in the instanton background is

$$J_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu + T_{\mu\nu}\,, \tag{4.10}$$

where $T_{\mu\nu}$ are the generators of an SO(9) subgroup of SO(16) gauge group. SO(9) is embedded into SO(16) in the manner so that the vector of SO(16) transforms as a spinor of SO(9); as in the previous subsection this completely specifies the SO(9) representation content of any SO(16) multiplet. Consequently, for example, a zero orbital angular momentum bound state of a vector with this instanton is fermionic. It transforms as a spinor under angular momentum and as a vector of the SO(16) gauge group.

The instanton of this subsection may be embedded into the natural or trivial SO(16) subgroup of SO(N) for $N \geq 16$, to yield the unique SO(N) instanton. All considerations of this subsection generalize to this case.

5. Applications in string theory

In the previous sections we have presented a construction of fermions out of bosons. This construction applies to any system that includes the appropriate non abelian gauge fields in the appropriate dimensions. In particular, it finds applications in perturbatively bosonic string theories (e.g. type-0A theory and the bosonic string), permitting the identification of nonperturbative fermions in these theories, as we describe in this section. In some examples we are able to identify the world sheet conformal field theory that corresponds to our construction and so are able to verify the fermionic nature of these excitations using world sheet techniques.

Gauge fields in perturbative string theory arise either from open strings on D-branes or from closed strings via compactification on group lattices. In subsection 5.1 and 5.2 we utilize the gauge fields on D-branes in bosonic string theories to construct fermions propagating on their world volume. In subsection 5.3 we utilize gauge fields from compactification to construct fermions propagating in space time.

In subsection 5.1 we study type-0A theory in the presence of $N{+}1$ $|\mathrm{D}p,+\rangle$-branes. The perturbative spectrum of both open and closed strings in this background is

[11] We thank E. Witten for discussions in this connection.

purely bosonic. According to the construction presented in section 4.1, the bound state of an instanton in the world volume theory of the N $|\mathrm{D}p,+\rangle$-branes, with a fundamental string stretching from the N-branes to the last remaining brane, is fermionic. However an instanton on a $|\mathrm{D}p,+\rangle$-brane is simply a $|\mathrm{D}0,+\rangle$-brane. Using this observation and world sheet techniques, it is easy to directly verify the fermionic nature of the corresponding system.

The general arguments of section 4 also predict that the same worldvolume configuration, namely the bound state of a string and an instanton on D-branes, is fermionic even in the bosonic string theory (the open string tachyon is approximately unsourced by an instanton that is large in string units). In the bosonic string, however, there seems no reason to associate an instantonic gauge field configuration with a D0-brane, or any other familiar worldsheet theory. Consequently we do not have an independent verification of the fermionic nature of this state.

In subsection 5.2 we consider type-0 theory; an orientifold of type-0B theory that includes 32 D9 and 32 anti D9-branes in its background. We consider the SO(32) instanton constructed out of the gauge fields on the D9-branes. It is natural to conjecture, in analogy with type-I theory, that this instanton represents the stable 0-brane of type-0 theory. According to the arguments of the previous section, the bound state of such an instanton with a bi-fundamental open string (a string stretching between a brane and an anti-brane) is fermionic. Using the conformal field theory description of the stable 0-brane, this prediction is easily verified.

In subsection 5.3 we point out that the constructions of section 4 imply that bosonic string theories compactified on group lattices have a potentially rich spectrum of fermions. We spell this out in a particularly interesting example; the bosonic string compactified on the SO(32) group lattice.

5.1 The D0-Dp system in type-0A theory

In this subsection we study a string theoretic implementation of the construction of fermions from SU(N) instantons presented in subsection 4.1. Using worldsheet techniques we find independent confirmation of the fermionic character of our state.

Consider $2^{\frac{p-2}{2}}+1$ $|\mathrm{D}p,+\rangle$ parallel branes, aligned along directions $x^1, x^2, \ldots, x^p$, in type-0A theory. Let the last of these branes be slightly separated, in transverse space, from the other $2^{\frac{p-2}{2}}$ branes, all of which are coincident. Consider an SU($2^{\frac{p-2}{2}}$) bundle with a single unit instanton charge on the world volume of the coincident branes. Such an instanton carries the charge of a single $|\mathrm{D}0,+\rangle$-brane [24], and so represents a $|\mathrm{D}0,+\rangle$-brane dissolved in the $2^{\frac{p-2}{2}}$ coincident $|\mathrm{D}p,+\rangle$-branes. Consequently, the quantum numbers of a string stretching from this instanton to the isolated $|\mathrm{D}p,+\rangle$ must be identical to those of a string running from a $|\mathrm{D}0,+\rangle$ to the isolated $|\mathrm{D}p,+\rangle$-brane.[12]

[12]Some aspects of the p-$p+4$ system of type-0B theory were studyed in [29].

It is easy to formally[13] verify that a $|\mathrm{D}0,+\rangle$-brane sitting on top of a $|\mathrm{D}p,+\rangle$, and connected to it with a fundamental string, is a spinor on the p spatial dimensional world volume of the brane. As we have remarked above (see subsection 3.2) strings between $|\mathrm{D}0,+\rangle$-brane and the isolated $|\mathrm{D}p,+\rangle$ are always in the Neveu-Schwarz sector. World sheet Neveu-Schwarz fermions in directions with Neumann-Dirichlet boundary conditions, $\psi^1, \psi^2, \ldots, \psi^p$ in this example, are always integer moded. Consequently 0-p open string states appear in degenerate multiplets obtained by quantizing the zero modes of the fermions $\psi^1, \psi^2, \ldots, \psi^p$, and so are SO($p$) spinors. As $0-p$ strings are oriented, these spinors are complex. The GSO projection, which retains only state of even worldsheet fermion number, projects out the odd chirality component states of this spinor, retaining a chiral SO(p) spinor, in precise agreement with the analysis of section 4.1.

5.2 The D-particle in type-0 theory

In this subsection we study a string theoretic implementation of the construction of fermions from SO(N) instantons presented in subsection 4.2. We verify the fermionic nature of our state using worldsheet techniques.

Recall that the stable D-particle of type-I theory, discovered by Sen [30], has the 'charge' of an SO(N) instanton residing on the background D9-branes of type-I theory [27]. type-0 theory possesses two stable D particles, $|\widehat{\mathrm{D}0},+\rangle$ and $|\widehat{\mathrm{D}0},-\rangle$, obtained by taking the orientifold projection of the two unstable D particles of 0B theory. As in [30], the action of the orientifold projects out the 0-0 tachyon, rendering these particles stable. It is natural to conjecture that these stable D-particles carry the 'charge' of SO(N) instantons on $|\mathrm{D}9,+\rangle$ and $|\mathrm{D}9,-\rangle$-branes respectively.

Consider type-0 theory with 32 $|\mathrm{D}9,+\rangle$-branes and 32 anti $|\mathrm{D}9,+\rangle$-branes in its background. The perturbative spectrum of both open and closed strings in this background is purely bosonic. According to the construction of section 4.2, an SO(32) instanton on the $|\mathrm{D}9,+\rangle$-branes, bound to a bifundamental 9-$\bar{9}$ string, is fermionic. As in the previous subsection (and according to the conjecture of the previous paragraph) the quantum numbers of such a bound state must be identical to those of the formal state obtained by quantizing a single $|\widehat{\mathrm{D}0},+\rangle$-anti $|\mathrm{D}9,+\rangle$ string. That is easy to verify. The relevant $(0,9)$ strings are always in the Neveu-Schwarz sector and have no GSO projection. The world sheet fermions ψ^i, $i = 1, \ldots, 9$ have Neumann-Dirichlet boundary conditions, and are consequently integer moded. Thus 0-9 string states appear in multiplets obtained by quantizing the zero modes of ψ^i, $i = 1, \ldots, 9$, and so are SO(9) spinors. Since these strings are unoriented the corresponding spinors are real.

[13] The rest of this subsection is formal as charge conservation forbids a single string from ending on a 0-brane, unless the later is dissolved in a higher dimensional brane, in which case the fundamental string charge is carried by lines of electric flux which spread out to infinity.

The fermions ψ^i also carry an anti 9-brane Chan-Paton index, and so are vectors[14] under SO(32), in precise agreement with the analysis of section 4.2.

5.3 Fermions in the bosonic string on the SO(32) group lattice

As we have remarked above, the construction of section 4 implies the existence of a spectrum of spacetime fermions in bosonic string theories compactified on U(N) or SO(N) group lattices. In this subsection we consider a special case; the bosonic string compactified down to 10 dimensions on the SO(32) group lattice. This particular compactification of the bosonic string seems especially interesting to us. Firstly, the left moving sector of this bosonic string is identical to the left moving sector of the SO(32) heterotic string. Secondly, in a speculative but intriguing conjecture, Bergman and Gaberdiel have suggested that this compactification of the bosonic string is S-dual to type-0 theory [22].

Apart from the metric, $B_{\mu\nu}$ the dilaton, and the uncharged tachyon, the low lying spectrum of this compactification includes SO(32) $\times$ SO(32) gauge bosons, a set of tachyons in the bi-fundamental of SO(32)$\times$SO(32), and the lattice deformation modes : massless scalars in the bi-adjoint representation of SO(32) $\times$ SO(32). The massive spectrum includes Hagedorn towers of states with each of these SO(32) $\times$ SO(32) quantum numbers.

Consider a nonperturbative field configuration in this theory that is characterized by the charge of a Z_2 instanton in one of the SO(32) gauge groups, as studied in section 4.2. According to section 4.2, the bound state of such an 'instanton' with, for instance, the bifundamental tachyons or massive string states, is fermionic. Consequently, this compactification of the bosonic string has a potentially rich spectrum of fermions.

Unfortunately we do not have a detailed understanding of any of the states of this subsection. However, one could speculate that tachyon condensation in this system has an endpoint, and that the nonperturbative fermions described in section become lighter as the tachyon condenses.

6. Discussion

We have generalized the Jackiw-Rebbi-Hasenfratz-'t Hooft construction of fermions from bosons, and argued that this construction may be used to identify a rich nonperturbative spectrum of fermions in perturbatively bosonic string theories. In some

[14] All 0-9 string states in the Neveu-Schwarz sector are massive (this follows from the zero point energy of these strings). Consequently, states appear in multiplets obtained by quantizing the zero modes of any one of the 32 possible strings, and so transform in the vector of SO(32). Strings in the Ramond sector are massless in their ground state. Consequently states in the Ramond sector appear in multiplets obtained by quantizing the zero modes of *all* 32 strings. In the Ramond sector only ψ^0 is integrally moded. The spinorial character of the type-I D particle follows by quantizing ψ^0 for all 32 0-9 strings. We thank S. Mukhi for discussions in this connection.

examples we are able to find an alternative worldsheet description of these states, and independently confirm their fermionic character.

Perhaps the observations in this paper make it a little more plausible that the bosonic string is connected to the M-theory mainland. In this connection we find the bosonic string compactified on the SO(32) lattice especially intriguing. As argued in section 5.3, this theory has a rich spectrum of 10 dimensional fermions. Further, its worldsheet theory is closely related to that of the SO(32) heterotic string. Indeed, it is possible to act on the conjectured relationship between 0A and IIA theories [3, 1, 2] with a sequence of very speculative dualities, to arrive at a conjectured relationship between the bosonic string theory on the SO(32) lattice and the SO(32) heterotic string. Wild as these ideas sound, we believe they are worth exploring.

Finally, at a more mundane level, it would also be interesting to address some of the unanswered technical questions touched upon in this paper. What are the rules that determine when D-branes are allowed to end on other D-branes in the bosonic string theory? What charges, if any, do these end points carry? What is the fate of fermions hosted on unstable D-branes after the branes have decayed into nothing? Is it possible to find a simple conformal field theory that describes a finite D-string stretching between two 3-branes?

Acknowledgments

We are grateful to D. Bak, O. Bergman, A. Dabholkar, R. Gopakumar, G. Mandal, J. Polchinski, T. Ramdas, S. Mukhi, F. Schaposnik, E. Silverstein, A. Strominger, D. Thompson, S. Trivedi, S. Wadia, E. Witten, and the participants of the Amsterdam Summer Workshop for useful discussions. We are especially grateful to Ashoke Sen for several extremely useful discussions and suggestions. S.M. would like to thank ITP Santa Barbara, TIFR Mumbai and the Amsterdam Summer Workshop for hospitality during various stages of completion of this paper. This work was supported in part by DOE grant DE-FG02-91ER40654, a Harvard Junior Fellowship and Fundacion Antorchas. The work of J.R.D. is supported by NSF grant PHY97-2202.

A. Charge-monopole angular momentum in the effective electrodynamics

Consider an SU(2) gauge theory (2.1) in the presence of a 't Hooft-Polyakov monopole with asymptotic Higgs vev v. Outside the monopole core ($r \gg 1/v$) and for processes of energy scale $\omega \ll v$ (2.1) is effectively a U(1) gauge theory. Consequently, as v is taken to infinity a U(1) description of the monopole (2.2) is arbitrarily good everywhere. Indeed, in a 'unitary gauge', in which the φ field is everywhere aligned along the 3 direction, (2.2) and (2.3) reduce

$$\begin{aligned}
A_0^a &= 0 \\
A_i^a &= \delta^{3a}\frac{(1-\cos(\theta))}{r}\widehat{\varphi} \\
\varphi^a &= v\delta^{a3}
\end{aligned} \tag{A.1}$$

the potential of a Dirac monopole of charge $g = \int_{S^2} \vec{B}^3.\vec{ds} = 4\pi$ (where $\vec{B}$ is the magnetic field) with a Dirac string along the $-\widehat{z}$ axis.

An SU(2) doublet consists of two particles of unbroken U(1) charge $\pm 1/2$. As $\frac{1}{2}\times 4\pi = 2\pi \times 1$, these particles saturate the Dirac quantization condition with the monopole (A.1). Now, according to Saha's classic calculation [31], a monopole of charge g and an electron of charge e separated in space by the position vector $\vec{r}$ have angular momentum

$$\vec{J} = \frac{eg}{4\pi}\widehat{r}. \tag{A.2}$$

Consequently the monopole (A.1) separated from either of the two particles from the SU(2) doublet by a position vector $\vec{r}$ has the half integral angular momentum $\vec{J} = \widehat{r}$, in agreement with the analysis of subsection 2.1.

We have demonstrated that the angular momentum of a monopole-fundamental bound state is half integral. An appeal to the spin- statistics theorem then demonstrates the fermionic character of these bound states. In fact, within an effective U(1) theory, Goldhaber has also given a remarkably simple direct argument for the fermionic statistics of such bound states [32].

B. The angular momentum of bound states

B.1 Angular momentum of the charge-monopole system in $3+1$ dimensions

As reviewed in section 2.1, Hasenfratz and 't Hooft have proposed that the angular momentum of a particle in the R^{th} representation of SU(2) in the presence of an SU(2) monopole is given by (2.4). As evidence for this proposal, Hasenfratz and 't Hooft have computed the angular momentum

$$L^a_{\text{total}} = L^a_{\text{particle}} + L^a_{\text{field}} = \epsilon^{abc}x^b(m\dot{x}^c) + 2\int d^3x\epsilon^{abc}\,\mathrm{Tr}\left(F_{0i}F_{ib}x^c + D_0\varphi x^c D_b\varphi\right) \tag{B.1}$$

of the configuration under consideration, in the classical limit. In their computation, the magnetic field F_{ib} and $D_b\varphi$ in (B.1) take their background values (2.2), while the electric field F_{0i} and $D_0\varphi$ are sourced by the charged particle, and are determined by the equations of motion. Thus (B.1) may be evaluated as a function of the particle's position in space x, and orientation in group space T^a, yielding [7] $L^a_{field} = T^a + e\epsilon^{abc} x^b A^k_c T^k$. Consequently,

$$L^a_{\text{total}} = L^a_{\text{particle}} + L^a_{\text{field}} = T^a + \epsilon^{abc} x^b p^c \,, \tag{B.2}$$

where $p^a = m\dot{x}^a + eA^b_a T^b$ is the momentum conjugate to x^a, confirming the identification of J^a with the total angular momentum of the system.

In subsection B.1 below we mimic the procedure outlined above to compute the angular momentum of a charged particle in the presence of a Yang-Mills instanton in 4+1 dimensions. We find that the answer is indeed given by (4.7), as predicted in subsection 4.1. We regard this computation as a check on the arguments of section 4.

B.2 Angular momentum of the charge instanton system in 4+1 dimensions

Consider the pure SU(2) gauge theory in 4+1 dimensions

$$S = -\frac{1}{4g^2_{YM}} \int d^5x F^a_{AB} F^{aAB} \,. \tag{B.3}$$

where A, B are spacetime indices that run from $0, 1, \ldots, 4$, and $a, b, \ldots$ are gauge indices that run from $1, 2, 3$. The 4 dimensional self dual instanton is a static solution of (B.3) and is given by

$$\begin{aligned} A^a_\mu &= 2\frac{\eta^{a\mu\nu} x^\nu}{x^2 + \lambda^2} \\ &= 2\eta^{a\mu\nu} x^\nu W \,, \\ F^a_{\mu\nu} &= 4\eta^{a\mu\nu} W (x^2 W - 1) \,. \end{aligned} \tag{B.4}$$

where $\mu, \nu, \ldots$ are spatial indices and take values from $1, \ldots, 4$. Here $W(x) = \frac{1}{x^2+\lambda^2}$, $x^2 = x^\mu x_\mu$ and λ is the scale of the instanton. $\eta^{a\mu\nu}$ is the 't Hooft symbol.

Consider a classical test particle with isospin charge vector T^a at the point y^μ. This creates a field A^a_0 according to the equation

$$D^{ab}_\mu F^b_{\mu 0} = g^2_{YM} T^a \delta^4(x - y) \,. \tag{B.5}$$

(recall that g^2_{YM} has dimensions length) which in turn gives rise to an angular momentum contribution given by

$$L_{\alpha\beta} = \frac{1}{g^2_{YM}} \int d^4x (x_\alpha F^a_{0\rho} F^a_{\rho\beta} - x_\beta F^a_{0\rho} F^a_{\rho\alpha}) \,. \tag{B.6}$$

Substituting the instanton configuration from (B.4), and performing simplifications which make use of the identities listed in Eqns A5-A15 of [28] we obtain

$$\begin{aligned} L_{\alpha\beta} = \int d^4x \Big[&(\eta^{a\beta\rho} \partial_\rho A^a_0 x^\alpha - \eta^{a\alpha\rho} \partial_\rho A^a_0 x^\beta) 4W(x^2 W - 1) - \\ &- (\eta^{a\beta\rho} x^\rho x_\alpha - \eta^{a\alpha\rho} x^\rho x_\beta) 16 W^2 (x^2 W - 1) \Big] \,. \end{aligned} \tag{B.7}$$

A covariant way to split these SO(4) generators into $SU(2)_R$ and $SU(2)_L$ is to project out the self-dual and anti-self dual parts of $L_{\alpha\beta}$ in (B.7). The self dual part is given by

$$\begin{aligned} J_{\alpha\beta} &= L_{\alpha\beta} + \frac{1}{2}\epsilon^{\alpha\beta\mu\nu} L_{\mu\nu} \\ &= \frac{1}{g_{YM}^2}\int d^4x \Big\{ \Big[\eta^{a\beta\mu}(x_\alpha \partial_\mu A_0^a - x_\mu \partial_\alpha A_0^a) - \eta^{a\alpha\mu}(x_\beta \partial_\mu A_0^a - x_\mu \partial_\beta A_0^a) - \\ &\qquad - \eta^{a\alpha\beta} x^\mu \partial_\mu A_0^a \Big] 4W(x^2W-1) + x^2 \eta^{a\alpha\beta} 16 A_0^a W^2 (x^2 W - 1) \Big\}. \end{aligned} \tag{B.8}$$

The anti-self dual part is given by

$$\begin{aligned} I_{\alpha\beta} &= L_{\alpha\beta} - \frac{1}{2}\epsilon^{\alpha\beta\mu\nu} L_{\mu\nu} \\ &= \frac{1}{g_{YM}^2}\int d^4x \Big\{ \Big[\eta^{a\beta\mu}(x_\alpha \partial_\mu A_0^a + x_\mu \partial_\alpha A_0^a) - \eta^{a\alpha\mu}(x_\beta \partial_\mu A_0^a + x_\mu \partial_\beta A_0^a) + \\ &\qquad + \eta^{a\alpha\beta} x^\mu \partial_\mu A_0^a \Big] 4W(x^2W-1) - \\ &\qquad - (2\eta^{a\beta\mu} x_\mu x_\alpha - 2\eta^{a\alpha\mu} x_\mu x_\beta + x^2 \eta^{a\alpha\beta}) 16 A_0^a W^2 (x^2 W - 1) \Big\}. \end{aligned} \tag{B.9}$$

The corresponding generators of $SU(2)_L$ are given by

$$K'_a = \frac{1}{4}\eta^{a\mu\nu} L_{\mu\nu} = \frac{1}{8}\eta^{a\mu\nu} J_{\mu\nu} \tag{B.10}$$

and those of $SU(2)_R$ are given by

$$L_a = \frac{1}{4}\bar{\eta}^{a\mu\nu} L_{\mu\nu} = \frac{1}{8}\bar{\eta}^{a\mu\nu} I_{\mu\nu}\,. \tag{B.11}$$

We wish to evaluate $J_{\alpha\beta}$ in (B.8). Note that the first two terms in (B.8) can be written as a total divergence, and vanish upon integrating. Consequently

$$J_{\alpha\beta} = \frac{1}{g_{YM}^2}\int d^4x \Big\{ -4\eta^{a\alpha\beta} x^\mu \partial_\mu A_0^a W(x^2W-1) + x^2 \eta^{a\alpha\beta} 16 A_0^a W^2 (x^2W-1) \Big\}. \tag{B.12}$$

The equation of motion for A_0, (B.5), may be simplified to

$$\partial_\mu \partial_\mu A_0^a + 4\epsilon^{abc}\eta^{b\mu\nu} x^\nu W \partial_\mu A_0^c - 8x^2 A_0^a W^2 = g_{YM}^2 T^a \delta^4(x-y)\,. \tag{B.13}$$

From (B.13) we solve for A_0^a as a function of T^a and derivatives of A_0^a, and substitute this solution into the second term of (B.12). Upon integrating by parts, all the terms in (B.12) involving A_0^a cancel, and we obtain the simple expression

$$J_{\alpha\beta} = (-2T^a y^2 W + 2T^a)\eta^{a\alpha\beta}\,. \tag{B.14}$$

The $\mathrm{SU}(2)_L$ generators are now given by

$$\begin{aligned} K'_a &= \frac{1}{8}\eta^{a\alpha\beta} J_{\alpha\beta}\,, \\ &= -T^a y^2 W + T^a\,, \\ &= \frac{1}{2}\eta^{a\mu\nu} x_\mu A^a_\nu T^a + T^a\,. \end{aligned} \tag{B.15}$$

In order to compute the total angular momentum, we add to K'_a the contribution due to the orbital angular momentum of the particle

$$K^{orb}_a = \frac{1}{4}\eta^{a\mu\nu}(x_\mu \dot{x}_\nu - x_\nu \dot{x}_\mu) = \frac{1}{2}\eta^{a\mu\nu} x_\mu \dot{x}_\nu \tag{B.16}$$

to obtain

$$K'^{\,\mathrm{total}}_a = \frac{1}{2}\eta^{a\mu\nu} x_\mu p_\nu + T^a\,, \tag{B.17}$$

where $p_\mu = m\dot{x}_\mu + A^a_\mu T^a$ is the canonical momentum conjugate to x^μ. Clearly (B.17) agrees with (4.7), and demonstrates the mixing of the $\mathrm{SU}(2)_L$ part of angular momentum with the SU(2) generators of the gauge group.

We now turn to the computation of $I_{\alpha\beta}$, the anti-self dual part of the conseved angular momentum. As above we use (B.13) to solve for A^a_0 in terms of T^a and its derivatives, and plug that solution into the last term in (B.9). After some tedious but straight forward integration by parts we find that all terms in (B.9) involving A_0 cancel, and

$$I_{\alpha\beta} = 2(2\eta^{a\beta\mu} y_\mu x_\alpha - 2\eta^{a\alpha\mu} x_\mu x_\beta + x^2 \eta^{a\alpha\beta}) T^a W\,. \tag{B.18}$$

It is easy to see that $I_{\alpha\beta}$ is the anti-self dual part of $(y_\alpha A^a_\beta - y_\beta A^a_\alpha)T^a$ where A^a is the instanton background. Thus the anti-self dual part of the total angular momentum including the anti-self dual part of the orbital angular momentum is given by

$$L_a = \frac{1}{2}\bar{\eta}^{a\mu\nu} x_\mu p_\nu\,, \tag{B.19}$$

in agreement with (4.7). Note that the anti-self dual part of the conserved angular momentum does not mix with the generators of the gauge group.

C. Supermultiplet structure of 3-string junctions in IIB theory

In type-IIB theory the 3 string junction of figure 1. is expected to appear in a $(1/4)^{\mathrm{th}}$ BPS multiplet of $\mathcal{N} = 4, d = 4$ supersymmetry. In this appendix we will construct this 3 string junction supermultiplet by quantizing the zero modes about the monopole of figure 2. We will work in the limit that the brane M approaches straight line joining A and B in figures 1 and 2. We will find that the expected supermultiplet structure emerges only upon assigning bosonic zero modes spin half, and fermionic zero modes spin zero, in agreement with the analysis of section 3. In this appendix we closely follow the arguments of section 6 of [18].

In the limit that the brane M approaches the straight line joining A and B in figure 2, the monopole (2.2) is marginally unstable to decay into two separate monopoles (in pictures, the D1-brane of figure 1 can split into two D strings, one from A to M, and the second from M to B). At this special point the monopole has 8 bosonic zero modes, which may be divided into

1. The centre of mass motions, which consist of the 3 centre of mass translations of the two monopoles, and a rotation of the overall U(1).

2. The relative motions which consist of the 3 relative separations of the two monopoles, and the rotation in the relative U(1).

Quantization of the U(1) rotations leads to states labeled by two electric charge integers $\{n_1, n_2\}$, where n_1 and n_2 are the number of quanta of flux travelling from M to A and from M to B respectively, in figure 1.

The bosonic zero modes have corresponding fermionic counterparts. The superpartners of the centre of mass motions are the goldstinos of the 8 supersymmetries broken by the monopole background. Quantizing these zero modes yields a $\mathcal{N} = 4$ vector multiplet, with 2^4 states. The remaining four fermionic zero modes create states with electric charge $\{\pm 1, 0\}$, $\{0, \pm 1\}$.

Consider a 3 string junction, as in figure 1, with k units of string charge leaving M. Such a junction may be created by exciting n_1 zero mode creation operators of charge $\{1, 0\}$ and n_2 zero mode creation operators of charge $\{0, 1\}$ such that $n_1 + n_2 = k$. It is easy to see that the number of such states is

1. $k + 1$ states built purely out of bosonic zero modes.

2. $2k$ states with 1 fermionic and $k - 1$ bosonic quanta.

3. $k - 1$ states built from two fermionic and $k - 2$ bosonic zero modes.

Tensoring this particle content with the $\mathcal{N} = 4$ vector multiplet yields a 'multiplet' with $2^6 k$ particles.

We expect these states to fill out a $(1/4)^{\text{th}}$ BPS multiplet of $\mathcal{N} = 4$ supersymmetry. This expectation is borne out if one assigns each bosonic zero mode spin half, and each fermionic zero mode spin zero. According to this assignment, the states described above constitute

1. A single spin $\frac{k}{2}$ multiplet.

2. Two spin $\frac{k-1}{2}$ multiplets.

3. A spin $\frac{k-2}{2}$ multiplet.

Tensoring with the $\mathcal{N} = 4$ vector multiplet yields the particle content of a $(1/4)^{\rm th}$ BPS multiplet with maximum spin $k/2 + 1$. This irreducible representation of the supersymmetry algebra may be identified with that constructed in [33] by tensoring the basic $(1/4)^{\rm th}$ BPS multiplet (obtained by quantizing the goldstinos of the 12 broken supercharges) with a spin $\frac{k-1}{2}$ representation of SU(2).

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